RJEŠENJE PISMENOG ISPITA, 30.06.2014.

A GRUPA

1. Zadani su vektori $\overrightarrow{AB} = 2\vec{m} - \vec{n}, \overrightarrow{BC} = \vec{m} + \vec{n}$. Odredite površinu paralelograma $ABCD$ ako je $|\vec{m}| = 1, |\vec{n}| = 4, \measuredangle(\vec{m}, \vec{n}) = \frac{\pi}{6}$.

Rješenje: $P = \left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$ jer je $\overrightarrow{BC} = \overrightarrow{AD}$.

$\overrightarrow{AB} \times \overrightarrow{BC} = (2\vec{m} - \vec{n}) \times (\vec{m} + \vec{n}) = 2\vec{m} \times \vec{m} + 2\vec{m} \times \vec{n} - \vec{n} \times \vec{m} - \vec{n} \times \vec{n} = 0 + 2\vec{m} \times \vec{n} + \vec{m} \times \vec{n} - 0 = 3\vec{m} \times \vec{n}$.

$P = |3\vec{m} \times \vec{n}| = |3| \cdot |\vec{m} \times \vec{n}| = 3 \cdot |\vec{m}| \cdot |\vec{n}| \cdot \sin \measuredangle(\vec{m}, \vec{n}) = 3 \cdot 1 \cdot 4 \cdot \sin\frac{\pi}{6} = 6.$

2. Zadana je funkcija $f(x) = $ arccos(-2x+3). Odredite domenu funkcije i prvu derivaciju u točki $x_0 = \frac{3}{2}$.

Rješenje: Uvjet na domenu je

$$-1 \leq -2x + 3 \leq 1 \implies 1 \leq x \leq 2 \implies \mathcal{D}_f = [1,2].$$

$$f'(x) = \frac{-1 \cdot (-2)}{\sqrt{1 - (-2x + 3)^2}}, f'\left(\frac{3}{2}\right) = 2.$$

3. Zadana je funkcija

$$f(x) = e^{\frac{x^2+3}{2x+2}}.$$

Odredite domenu, intervale monotonosti te lokalne ekstreme funkcije.

Rješenje: $2x + 2 \neq 0 \Rightarrow x \neq -1$ pa je $\mathcal{D}_f = \mathbb{R}\backslash\{-1\}$

$$f'(x) = e^{\frac{x^2+3}{2x+2}} \cdot \frac{2x(2x+2) - (x^2+3)2}{(2x+2)^2} = e^{\frac{x^2+3}{2x+2}} \cdot \frac{2x^2 + 4x - 6}{(2x+2)^2} = e^{\frac{x^2+3}{2x+2}} \cdot \frac{2(x-1)(x+3)}{(2x+2)^2}$$

$f'(x) = 0 \implies x_1 = 1, x_2 = -3$ (stacionarne točke)

	$\langle -\infty, -3\rangle$	$\langle -3, -1\rangle$	$\langle -1, 1\rangle$	$\langle 1, +\infty\rangle$
f'	+	-	-	+
f	raste	pada	pada	raste

Lokalni ekstremi su $T_{max} = (-3, e^{-3}), T_{min} = (1, e)$.

4. Riješite integral

$$\int (2x - 1) \ln 4x \, dx.$$

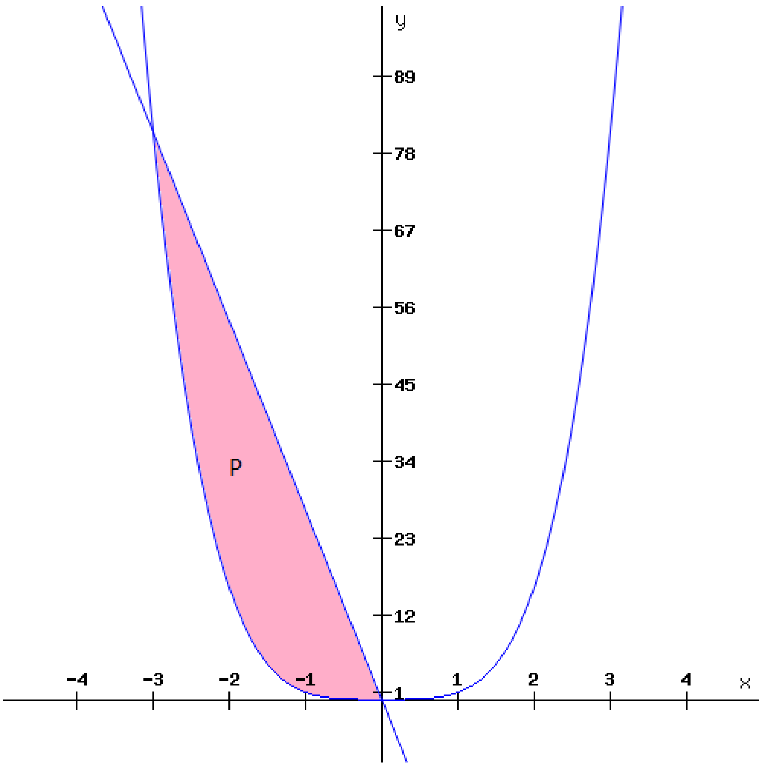
Rješenje: metoda parcijalne integracije: $\begin{bmatrix} u = \ln 4x & v' = (2x-1) \\ u' = \frac{1}{x} & v = x^2 - x \end{bmatrix}$

$$\int (2x-1)\ln 4x \, dx = uv - \int u'v = \ln 4x \, (x^2 - x) - \int \frac{x^2 - x}{x} dx = \ln 4x \, (x^2 - x) - \int (x - 1) \, dx$$
$$= \ln 4x \, (x^2 - x) - \frac{x^2}{2} + x + c$$

5. Izračunajte površinu omeđenu grafom funkcije $y = x^4$ i pravcem $y + 27x = 0$.

Rješenje: Sjecište funkcije $y = x^4$ i pravca $y + 27x = 0$ nalazi se izjednačavanjem $y = x^4$ i $y = -27x$.

$$\begin{aligned} x^4 &= -27x \\ x^4 + 27x &= 0 \quad \Longrightarrow \quad x_1 = 0, x_2 = -3 \\ x(x^3 + 27) &= 0 \end{aligned}$$



$$P = \int_{-3}^{0} (-27x - x^4)dx = -27\frac{x^2}{2} - \frac{x^5}{5}\Big|_{-3}^{0} = 0 + 27 \cdot \frac{9}{2} + \frac{(-3)^5}{5} = \frac{729}{10}$$

B GRUPA

1. Zadani su vektori $\overrightarrow{AB} = \vec{m} - 3\vec{n}, \overrightarrow{BC} = 2\vec{m} + \vec{n}$. Odredite površinu paralelograma $ABCD$ ako je $|\vec{m}| = 2, |\vec{n}| = 3, \measuredangle(\vec{m}, \vec{n}) = \frac{\pi}{3}$.

Rješenje: $P = |\overrightarrow{AB} \times \overrightarrow{BC}|$ jer je $\overrightarrow{BC} = \overrightarrow{AD}$.

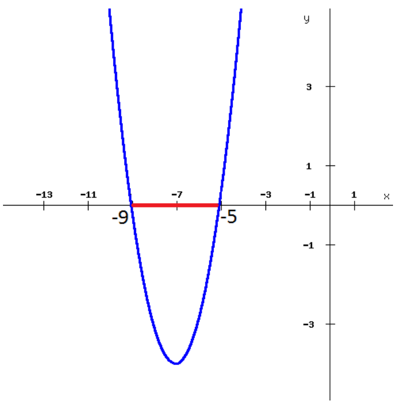
$$\overrightarrow{AB} \times \overrightarrow{BC} = (\vec{m} - 3\vec{n}) \times (2\vec{m} + \vec{n}) = 2\vec{m} \times \vec{m} + \vec{m} \times \vec{n} - 6\vec{n} \times \vec{m} - 3\vec{n} \times \vec{n} = 0 + \vec{m} \times \vec{n} - 6\vec{m} \times \vec{n} - 0$$
$$= 7\vec{m} \times \vec{n}.$$

$$P = |7\vec{m} \times \vec{n}| = |7| \cdot |\vec{m} \times \vec{n}| = 7 \cdot |\vec{m}| \cdot |\vec{n}| \cdot \sin\measuredangle(\vec{m}, \vec{n}) = 7 \cdot 2 \cdot 3 \cdot \sin\frac{\pi}{3} = 21\sqrt{3}.$$

2. Zadana je funkcija $f(x) = \sqrt{-x^2 - 14x - 45}$. Odredite domenu funkcije i prvu derivaciju u točki $x_0 = -6$.

Rješenje: Uvjet na domenu je

$$\begin{array}{c} -x^2 - 14x - 45 \geq 0 \\ x^2 + 14x + 45 \leq 0 \\ (x+5)(x+9) \leq 0 \end{array} \implies \mathcal{D}_f = [-9, -5].$$



$$f'(x) = \frac{-2x - 14}{2\sqrt{-x^2 - 14x - 45}} = \frac{-(x+7)}{\sqrt{-x^2 - 14x - 45}}, \quad f'(-6) = \frac{-1}{\sqrt{3}}.$$

3. Zadana je funkcija

$$f(x) = e^{\frac{x^2+2}{2x+1}}.$$

Odredite domenu, intervale monotonosti te lokalne ekstreme funkcije.

Rješenje: $2x + 1 \neq 0 \Rightarrow x \neq \frac{-1}{2}$ pa je $\mathcal{D}_f = \mathbb{R} \setminus \left\{-\frac{1}{2}\right\}$

$$f'(x) = e^{\frac{x^2+2}{2x+1}} \cdot \frac{2x(2x+1) - (x^2+2)2}{(2x+1)^2} = e^{\frac{x^2+2}{2x+1}} \cdot \frac{2x^2 + 2x - 4}{(2x+1)^2} = e^{\frac{x^2+2}{2x+1}} \cdot \frac{2(x-1)(x+2)}{(2x+1)^2}$$

$f'(x) = 0 \implies x_1 = 1, x_2 = -2$ (stacionarne točke)

	$\langle -\infty, -2 \rangle$	$\langle -2, -\frac{1}{2} \rangle$	$\langle -\frac{1}{2}, 1 \rangle$	$\langle 1, +\infty \rangle$
f'	+	-	-	+
f	raste	pada	pada	raste

Lokalni ekstremi su $T_{max} = (-2, e^{-2}), T_{min} = (1, e)$.

4. Riješite integral

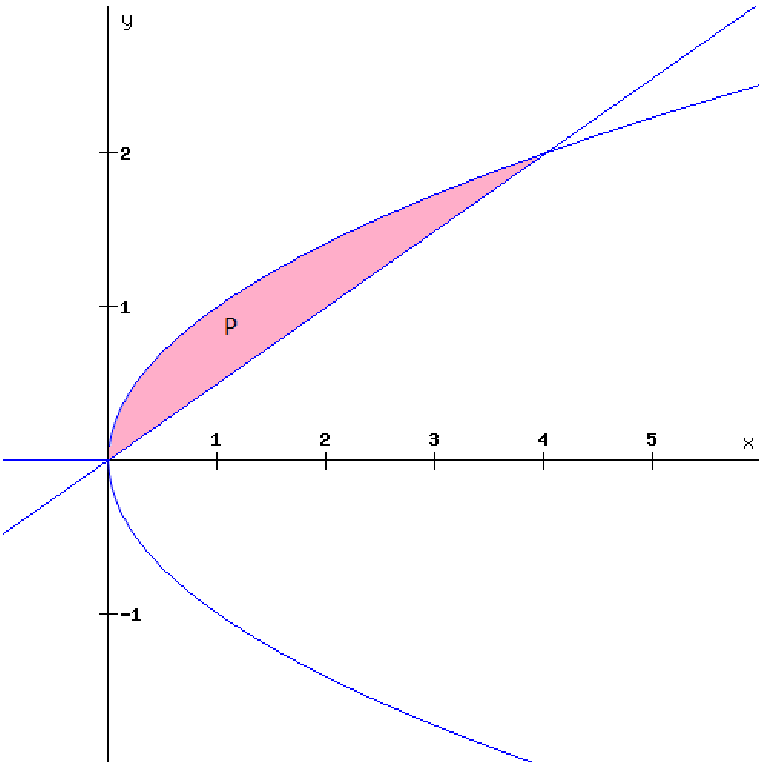
$$\int (2x+1)\sin 2x\, dx.$$

Rješenje: metoda parcijalne integracije: $\begin{bmatrix} u = 2x+1 & v' = \sin 2x \\ u' = 2 & v = -\frac{1}{2}\cos 2x \end{bmatrix}$

$$\int (2x+1)\sin 2x\, dx = uv - \int u'v = -\frac{1}{2}(2x+1)\cos 2x + \int \cos 2x\, dx = -\frac{1}{2}(2x+1)\cos 2x + \frac{1}{2}\sin 2x + c$$

5. Izračunajte površinu omeđenu grafom funkcije $x = y^2$ i pravcem $y - \frac{1}{2}x = 0$.

Rješenje: Sjecište funkcije $x = y^2$ i pravca $y - \frac{1}{2}x = 0 \implies \begin{matrix} y = \frac{1}{2}y^2 \\ \frac{1}{2}y^2 - y = 0 \\ y\left(\frac{1}{2}y - 1\right) = 0 \end{matrix} \implies \begin{matrix} y_1 = 0 & x_1 = 0 \\ y_2 = 2 & x_2 = 4 \end{matrix}$



$$P = \int_0^4 \left(\sqrt{x} - \frac{1}{2}x\right) dx = \left(\frac{2}{3}\sqrt{x^3} - \frac{1}{4}x^2\right)\Big|_0^4 = \frac{16}{3} - 4 = \frac{4}{3}.$$