DOMENA FUNKCIJE

Domena (prirodno područje definicije) funkcije je skup $D_f = \{x \in \mathbb{R} \; : \; f(x) \in \mathbb{R}\}$.
Uvjeti za traženje domene funkcije:

1. $f(x) = \dfrac{g(x)}{h(x)} \quad \Rightarrow \quad h(x) \neq 0$ (uvjet nazivnika)
2. $f(x) = \sqrt[2n]{g(x)} \quad \Rightarrow \quad g(x) \geq 0$ (uvjet parnog korijena)
3. $f(x) = \log_a(g(x)) \quad \Rightarrow \quad g(x) > 0$ (uvjet logaritma)
4. $f(x) = \arcsin(g(x))$ ili $f(x) = \arccos(g(x)) \quad \Rightarrow \quad -1 \leq g(x) \leq 1$
5. $f(x) = \operatorname{cth}(g(x)) \quad \Rightarrow \quad g(x) \neq 0$
6. $f(x) = \operatorname{Arch}(g(x)) \quad \Rightarrow \quad g(x) \geq 1$
   $f(x) = \operatorname{Arth}(g(x)) \quad \Rightarrow \quad -1 < g(x) < 1$
   $f(x) = \operatorname{Arcth}(g(x)) \quad \Rightarrow \quad g(x) < -1$ ili $g(x) > 1$

ASIMPTOTE

- pravac $x = a$ je **vertikalna (okomita) asimptota** funkcije $y = f(x)$ ako je:

$$\lim_{x \to a^+} f(x) = \pm\infty \quad \text{i/ili} \quad \lim_{x \to a^-} f(x) = \pm\infty.$$

  Za točku a uzimamo točke koje nisu u domeni funkcije (točke prekida funkcije) ili točke na rubu otvorenog intervala domene.

- pravac $y = b$ je **horizontalna (vodoravna) asimptota** funkcije $y = f(x)$ ako je:
  - $\lim\limits_{x \to +\infty} f(x) = b$ (desna hor. as.)
  - $\lim\limits_{x \to -\infty} f(x) = b$ (lijeva hor. as.)

- pravac $y = kx + l$ je **desna kosa asimptota** funkcije $y = f(x)$ gdje je:

$$k = \lim_{x \to +\infty} \frac{f(x)}{x}, \quad l = \lim_{x \to +\infty} (f(x) - kx)$$

  pravac $y = kx + l$ je **lijeva kosa asimptota** funkcije $y = f(x)$ gdje je:

$$k = \lim_{x \to -\infty} \frac{f(x)}{x}, \quad l = \lim_{x \to -\infty} (f(x) - kx)$$

TABLICA DERIVACIJA

$c' = 0, \quad c \in \mathbb{R}$	$x' = 1$
$(x^n)' = nx^{n-1}, \quad n \in \mathbb{Z}$	$(x^c)' = cx^{c-1}, \quad c \in \mathbb{R}, x > 0$
$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$	$(\sqrt{x})' = \frac{1}{2\sqrt{x}}, \quad x > 0$
$(a^x)' = a^x \ln a, \quad a > 0$	$(\log_a x)' = \frac{1}{x \ln a}, \quad a > 0, a \neq 1, x > 0$
$(e^x)' = e^x$	$(\ln x)' = \frac{1}{x}, \; x > 0$
$(\sin x)' = \cos x$	$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad \lvert x\rvert < 1$
$(\cos x)' = -\sin x$	$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}, \quad \lvert x\rvert < 1$
$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$	$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$
$(\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$	$(\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$
$(\operatorname{sh} x)' = \operatorname{ch} x$	$(\operatorname{Arsh} x)' = \frac{1}{\sqrt{1+x^2}}$
$(\operatorname{ch} x)' = \operatorname{sh} x$	$(\operatorname{Arch} x)' = \frac{1}{\sqrt{x^2-1}}, \quad x > 1$
$(\operatorname{th} x)' = \frac{1}{\operatorname{ch}^2 x}$	$(\operatorname{Arth} x)' = \frac{1}{1-x^2}, \quad \lvert x\rvert < 1$
$(\operatorname{cth} x)' = -\frac{1}{\operatorname{sh}^2 x}$	$(\operatorname{Arcth} x)' = \frac{1}{1-x^2}, \quad \lvert x\rvert > 1$

DERIVACIJE - DODATNO

- $(f(x) \pm g(x))' = f'(x) \pm g'(x)$
- $(c \cdot f(x))' = c \cdot f'(x)$
- $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
- $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g(x)^2}$
- $\Big((f \circ g)(x)\Big)' = \Big(f(g(x))\Big)' = f'(g(x)) \cdot g'(x)$
- **jednadžba tangente** na graf funkcije f u točki $T(x_0, f(x_0))$: $y - f(x_0) = f'(x_0)(x - x_0)$

TABLICA INTEGRALA

$$\int \mathrm{d}x = x + C$$

$$\int x^r \,\mathrm{d}x = \frac{x^{r+1}}{r+1} + C, \quad r \in \mathbb{R}, r \neq -1$$

$$\int \frac{\mathrm{d}x}{x} = \ln|x| + C$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$$\int a^x \,\mathrm{d}x = \frac{a^x}{\ln a} + C, \quad a > 0, a \neq 1$$

$$\int \operatorname{ch} x \,\mathrm{d}x = \operatorname{sh} x + C$$

$$\int \operatorname{sh} x \,\mathrm{d}x = \operatorname{ch} x + C$$

$$\int \cos x \,\mathrm{d}x = \sin x + C$$

$$\int \sin x \,\mathrm{d}x = -\cos x + C$$

$$\int \frac{\mathrm{d}x}{\cos^2 x} = \operatorname{tg} x + C$$

$$\int \frac{\mathrm{d}x}{\sin^2 x} = -\operatorname{ctg} x + C$$

$$\int \frac{\mathrm{d}x}{\operatorname{ch}^2 x} = \operatorname{th} x + C$$

$$\int \frac{\mathrm{d}x}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1-x^2}} = \arcsin x + C$$

$$\int \frac{\mathrm{d}x}{1+x^2} = \operatorname{arctg} x + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{1+x^2}} = \operatorname{Arsh} x + C = \ln\left(x + \sqrt{1+x^2}\right) + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2-1}} = \operatorname{Arch} x + C = \ln\left|x + \sqrt{x^2-1}\right| + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C, \quad a > 0$$

$$\int \frac{\mathrm{d}x}{a^2+x^2} = \frac{1}{a}\operatorname{arctg} \frac{x}{a} + C, \quad a > 0$$

$$\int \frac{\mathrm{d}x}{\sqrt{a^2+x^2}} = \ln\left(x + \sqrt{a^2+x^2}\right) + C, \quad a > 0$$

$$\int \frac{\mathrm{d}x}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + C, \quad a > 0$$

$$\int \frac{\mathrm{d}x}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{x+a}{x-a}\right| + C, \quad a > 0$$

$$\int \frac{\mathrm{d}x}{\sin x} = \ln\left|\operatorname{tg} \frac{x}{2}\right| + C$$

$$\int \frac{\mathrm{d}x}{\cos x} = \ln\left|\operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C$$