Postoptimal Analysis in Intelligent Sea Transportation System Optimization

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Abstract: - A procedure for the ship dynamic positioning postoptimal analysis in Intelligent Sea Transportation System Optimization is proposed. The dynamic positioning control system design is based on the optimal constrained covariance control (OC^3). In that way some disadvantages of the classical optimal control technique are avoided. The presented numerical example illustrates new concept of the proposed approach.

Key-Words: - Intelligent Transportation System, Dynamic Positioning, LQG control, Optimization problems, Postoptimal analysis

1 Introduction

Dynamic Positioning of floating vessels is a technique for maintaining the position and heading of the vessel without the use of mooring system [1]. The basic forces and motions are presented in Fig. 1. Today this maneuver technique is very important for various logistics facility. A Mobile Offshore Base (MOB) Project as a large floating platform is well-known example [2]. Classical approach in dynamic positioning control system design is Linear Quadratic Gaussian (LQG) optimal control procedure. The main disadvantage of that procedure is selection problem of matrices Q and R (there is not physical sense). The main motivation for use Optimal Constrained Covariance Control Theory (OC^3) is that many real control systems have performance requirements naturally stated in the terms of the root-mean-square (RMS) values [3]. These requirements are usually given in the form of inequality constraints. The optimal control problem is characterized by compromises and tradeoffs, with performance requirements and magnitude of the input energy. For example, the objective of a dynamic positioning system is to maintain the position and heading of a vessel at reference values with acceptable accuracy. The design of the systems involves a compromise between the accuracy of holding a position and the need to suppress excessive thruster response.

The use of Covariance Control Theory by the procedure of assigning the state covariance has a theoretical meaning only [4]. Namely, the assigning of a complete covariance matrix is a very hard requirement in a lot of real engineering systems. Apart from this, the procedure for assigning desired cross-correlation terms is usually unwieldy (particularly for large-order systems). The requirements in the form of inequality constraints (some of diagonal terms) are more acceptable. The overcome some problems the use of the constrained LQG Control is proposed [5]. The non-existing proof of the convergence is the main disadvantage of this method. The suggested procedure gives only the local optimality condition. Using the technique presented in [6] a number of above disadvantages are avoided. The optimal linear controller is designed with OC3 technique in such a way that the specified state covariance of a closed loop system is below the ordered ones. This is achieved with minimum input energy. Next, the suggested procedure holds the original convexity of LQR problem.

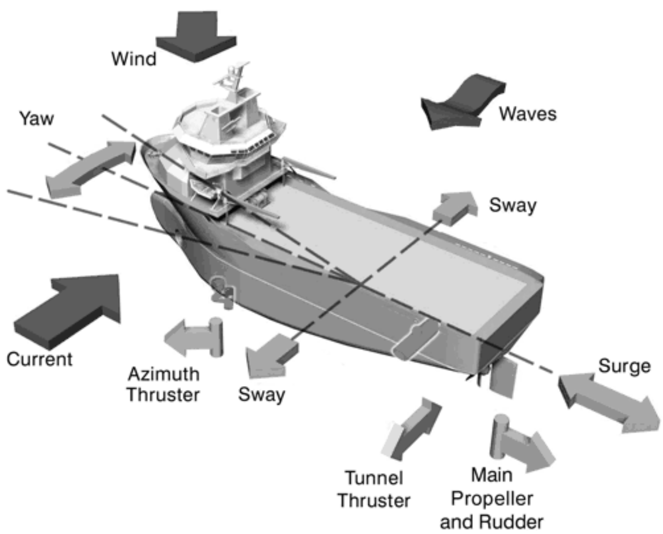

Fig. 1. Dynamic Positioning System

2 Problem Formulation

Consider the continuous linear system described by:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{Ax}(t) + \mathbf{Bu}(t) + \mathbf{Gw}(t)\\ \mathbf{y}(t) &= \mathbf{Cx}(t) + \mathbf{v}(t)\end{aligned} \tag{1}$$

where $\mathbf{x}$ is the n-dimensional state vector, $\mathbf{u}$ is the p-dimensional input vector, $\mathbf{y}$ is the r-dimensional output vector, and $\mathbf{w}$ and $\mathbf{v}$ are Gaussian white noise with zero mean and covariance matrices $\mathbf{R}_w$ and $\mathbf{R}_v$, respectively. The required performances are given in the form of inequality constraints:

$$\mathrm{diag}(\mathbf{D}_x) \le \mathbf{d_0} \tag{2}$$

where $\mathbf{D_X}$ is the state covariance matrix of closed loop system and $\mathbf{d_0}$ are desired upper limits for diagonal elements of $\mathbf{D_X}$. The cost function (price) is given in the form:

$$J = \mathrm{trace}(\mathbf{RD_u}) \tag{3}$$

where $\mathbf{D_u}$ is the control input covariance matrix of closed loop system and $\mathbf{R}$ is weighting matrix. $\mathbf{D_X}$ and $\mathbf{D_u}$ are defined as:

$$\begin{aligned}\mathbf{D_x} &= \mathrm{E}\left[\mathbf{x}(t)\mathbf{x}(t)^{\mathrm{T}}\right]\\ \mathbf{D_u} &= \mathrm{E}\left[\mathbf{u}(t)\mathbf{u}(t)^{\mathrm{T}}\right]\end{aligned} \tag{4}$$

The optimal regulator has the form:

$$\begin{aligned}\dot{\hat{\mathbf{x}}} &= \mathbf{A}\hat{\mathbf{x}} + \mathbf{Bu} + \mathbf{K_f}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})\\ \mathbf{u} &= -\mathbf{K_r}\hat{\mathbf{x}}\end{aligned} \tag{5}$$

where $\hat{x}$ is optimal estimation of $\mathbf{x}$, $\mathbf{K}_f$ is Kalman filter gain and $\mathbf{K}_r$ is full-order static controller. The solution of the Kalman filter equation gives the stationary state estimation error covariance matrix in the form:

$$\mathbf{P}_{\tilde{\mathbf{x}}} = \mathrm{E}[\mathbf{x}(t) - \hat{\mathbf{x}}(t)][\mathbf{x}(t) - \hat{\mathbf{x}}(t)]^{\mathrm{T}} \tag{6}$$

The estimation error is uncorrelated with any estimate of the state. The required performances are transformed in the new form of inequality constraints:

$$\mathrm{diag}(\mathbf{D}_{\hat{\mathbf{x}}} + \mathbf{P}_{\tilde{\mathbf{x}}}) \le \mathbf{d_0} \tag{7}$$

Now, we define weighting matrix $\mathbf{Q}$ as:

$$\mathbf{Q} = \mathbf{XX}^{\mathrm{T}} \tag{8}$$

where $\mathbf{X}$ is an arbitrary matrix and $\mathbf{Q}$ is always symmetric and positive semi-definite matrix. The terms of $\mathbf{X}$ are variables in the considered optimization problem. In that way, a local minimizer is defined as the well-known LQR problem:

$$\mathbf{K_r} = \mathbf{LQR}(\mathbf{A}, \mathbf{B}, \mathbf{Q}, \mathbf{R}) \tag{9}$$

Since innovations signal is a white noise process with mean zero and covariance $\mathbf{R}_v$ which is independent of $\mathbf{x}$ the stationary covariance matrices of the estimate and the input can be computed from:

$$\begin{aligned}(\mathbf{A} - \mathbf{BK_r})\mathbf{D}_{\hat{\mathbf{x}}} + \mathbf{D}_{\hat{\mathbf{x}}}(\mathbf{A} - \mathbf{BK_r})^{\mathbf{T}} + \mathbf{K_f R_v K_f^T} &= \mathbf{0}\\ \mathbf{D_u} &= \mathbf{K_r D_{\hat{x}} K_r^T}\end{aligned} \tag{10}$$

In this way, the algorithm is based on solving a sequence of standard linear quadratic control problems. This can be done with the algorithm shown in Fig 2.

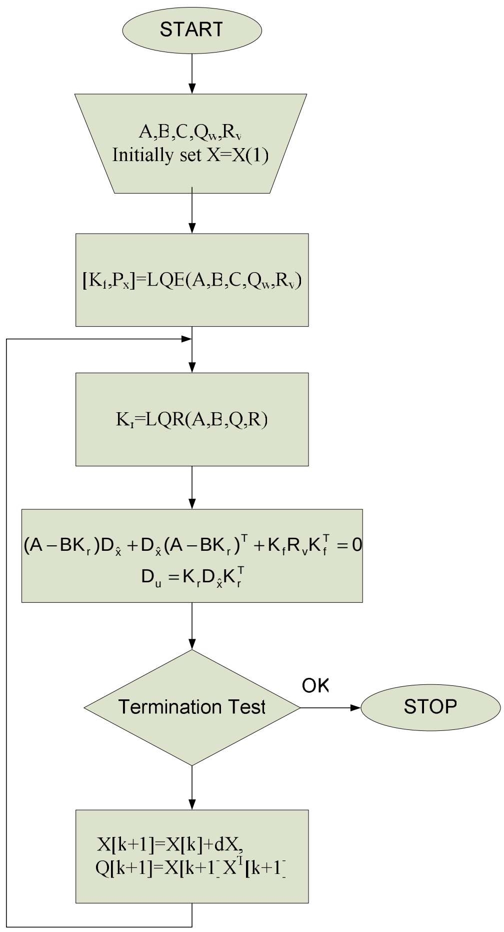

Fig. 2. OC[3] Algorithm

Without proof, it is clear that the suggested procedure holds the original convexity of the LQR problem. There is no problem to supply the SQP algorithm with the analytically defined gradients of the cost function (3) and constraints (7). Some characteristics of dynamic positioning control design are described in [3]. There is a notable difference between the standard LQG procedure and constrained LQG. The standard LQG solution is not dependent of the characteristics of the disturbances (w(t) and v(t)). The constrained LQG procedure takes into account both the process disturbances and the estimation solution.

3 Postoptimal Analysis

When the optimization procedure is finished, the sensitivity of the solutions to desired system performances, model inaccuracies and other initial conditions have to be analyzed. This analysis is known as postoptimal analysis [7]. When the sensitivity of solutions to desired system performances is of our concern, then it can be shown that under particular circumstances, a slight change of desired system performances could significantly improve the optimal solution value. Namely, a slight relaxation of desired system position accuracy could result with significant energy savings. As part of postoptimal analysis, the possibilities of price-performance (cost-effectiveness) improvements can be tested. The optimization problem can be given in the general form by:

$$\begin{aligned} &\min_{\mathbf{x}} f^0(\mathbf{x}) \\ &f^i(\mathbf{x}) \le q_i \qquad i = 1,\ldots,p \end{aligned} \tag{11}$$

where:

$f^0(\mathbf{x})$ - cost (price) function,
$f^i(\mathbf{x})$ - constraint function,
θ_i - constraint (performance) value.

The above optimization is easy to explain. The cost function represents the price of realization (such as energy consumption), while the constraint function represents the desired technical performances of our system (such as desired position accuracy). The corresponding augmented Lagrange function is:

$$L(\mathbf{x},\lambda,\theta) = f^0(\mathbf{x}) + \sum_{i=1}^{p} \lambda_i (f^i(\mathbf{x}) - \theta_i) \tag{12}$$

Assuming that Slater's condition [7] is valid for some point $\mathbf{x}^*$ and Θ^*, then:

$$\frac{\partial f^0(\mathbf{x}^*,\theta^*)}{\partial \theta_i^*} = -\lambda_i^* \tag{13}$$

Equation (13) can be interpreted as the shadow price. This term is often used in economics when the optimal solutions are sought. Equation (13) gives the relation for the sensitivity of solution to small change of constrained value (11). A small (or zero) value of Lagrange multiplier indicates that a slight change in this constraint does not have influence on the cost function. On the other hand, a large value of Lagrange multiplier indicates that the corresponding optimal value of the cost function is more susceptible to changes in this constraint. In the case of OC^3 design, equation (13) expresses the cost sensitivity related to the slight change of control system accuracy performances. However, sometimes the normed equation (13) is preferred, and is given by:

$$s_i = \frac{\dfrac{\partial f^0(\mathbf{x}^*,\theta^*)}{f^0(\mathbf{x}^*,\theta^*)}}{\dfrac{\partial \theta_i^*}{\theta_i^*}} = -\lambda_i^* \frac{\theta_i^*}{f^0(\mathbf{x}^*,\theta^*)} \tag{13}$$

Here the relative change of the cost function optimal value and constrained values are used. Parameter s_i represents the normed shadow price.

4 Example

The proposed method of a postoptimal analysis is applied to the dynamic positioning of the floating vessel, given in [8]. Only the sway motion is analyzed. The LF subsystem of mathematical model is given by:

$$A = \begin{bmatrix} -0.0546 & 0 & 0.5435 \\ 1 & 0 & 0 \\ 0 & 0 & -1.55 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1.55 \end{bmatrix} \qquad G = \begin{bmatrix} 0.5435 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

$$Q_w = 5.1984 * 10^{-6} \qquad R_v = 10^{-5}$$

The response of the floating vessel on random disturbance is presented in Fig. 3.

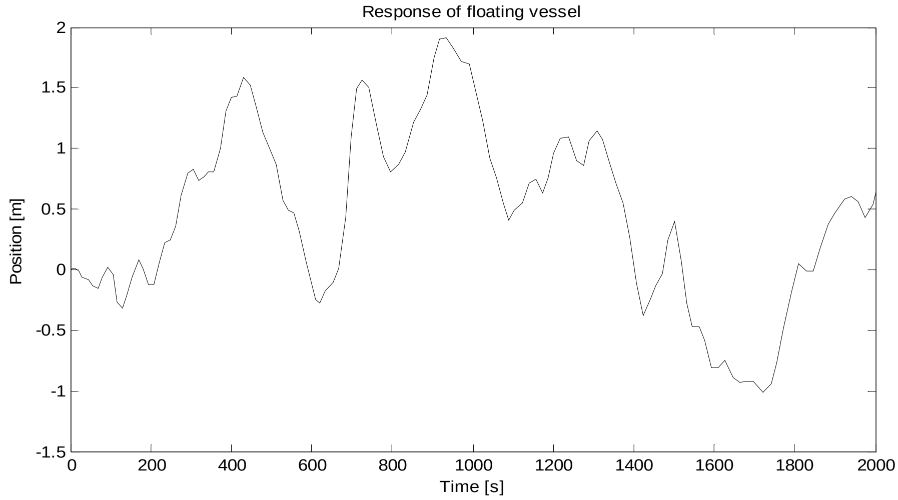

Fig. 3. Response of floating vessel to random disturbance

Kalman filter gain matrix (**K**$_f$) and the corresponding error covariance matrix for LF subsystem ($\mathbf{P}_{\tilde{x}}$) are:

$$K_f = \begin{bmatrix} 0.3464 \\ 0.8324 \\ 0 \end{bmatrix}$$

$$P_{\tilde{x}} = \begin{bmatrix} 0.3073 & 0.3464 & 0 \\ 0.3464 & 0.8324 & 0 \\ 0 & 0 & 0 \end{bmatrix} * 10^{-5}$$

The cost function in the postoptimal analysis was chosen to represent the total energy consumption, while the constraint function represents the positioning error. The results of the postoptimal analysis are given in Fig. 3. It can be seen from Fig.4. that the (shadow price) parameter s_i is approximately one until the positioning error dispersion becomes 2 meters. After that the shadow price value steeply rises. The interpretation of this example from the economic aspect is that there is the price to be paid if we insist to have the positioning accuracy better than 2 meters. Subsequent techno-economical analysis must establish justification for accuracy improvement below 2 meters.

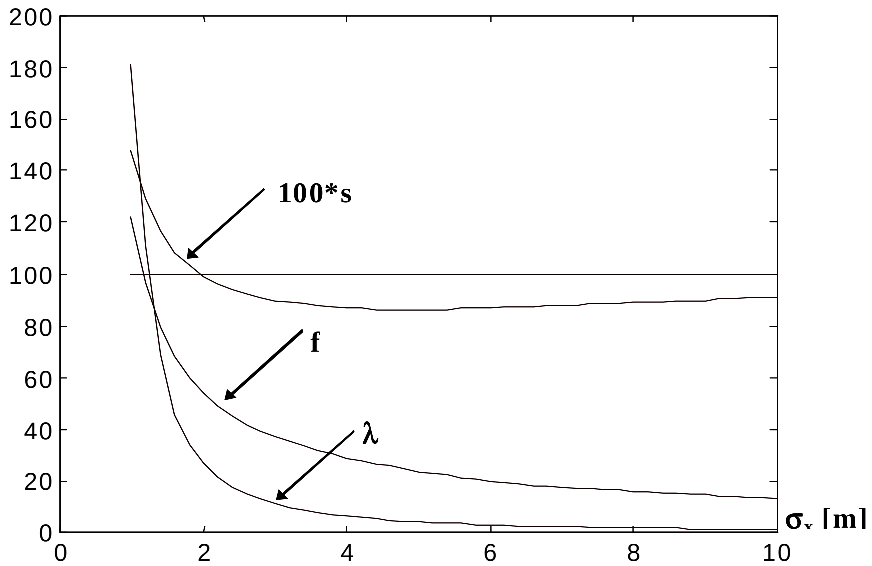

Fig. 4. Shadow price function

5 Conclusion

The main motivation for use Optimal Constrained Covariance Control Theory (OC3) is that many real control systems have performance requirements naturally stated in the terms of the root-mean-square (RMS) values. These requirements are usually given in the form of inequality constraints. The optimal control problem is characterized by compromises and tradeoffs, with performance requirements and magnitude of the input energy. For example, the objective of a dynamic positioning system is to maintain the position and heading of a vessel at reference values with acceptable accuracy. The design of the systems involves a compromise between the accuracy of holding a position and the need to suppress excessive thruster response.

When the sensitivity of solutions to desired system performances is of our concern, then it can be shown that under particular circumstances, a slight change of desired system performances could significantly improve the optimal solution value. Namely, a slight relaxation of desired system position accuracy could result with significant energy savings.

The proposed method provides a means for the analysis of desired performance, set by the designer, for the intelligent control system, according to the total cost (energy consumption). Sometimes it can be concluded that a slight relaxation of desired accuracy specifications (if technically sound) can result in significant total energy savings. Future research should investigate interdependence between parameters of shadow price and robustness of the control system. A preliminary analysis shows that some form of interdependence exists, because with significant growth of the shadow price, the robustness of the control system deteriorates.

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